CHATTERING ANALYSIS OF THE SYSTEM WITH HIGHER ORDER SLIDING MODE CONTROL

THESIS

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Abstract

Sliding mode control methodology (SMC) is considered as an efficient technique in several aspects for control systems. Nevertheless, applying first-order sliding mode control method in real-life has what so-called chattering phenomenon. Chattering is a high frequency movement that makes the state trajectories quickly oscillating around the sliding surface. This phenomenon may lead to degrade the system effectiveness, or even worst it may lead to fast damage of mechanical parts of the system. Presently, the major solutions to this drawback are asymptotic observers and high-order sliding mode control. The higher order sliding mode control is an expansion of the conventional sliding mode, and it can cancel the imperfection of the conventional sliding mode methodology and sustain its advantages. Since the second-order sliding mode controller, for example, super twisting algorithm, has unsophisticated construction and requires a lesser amount of acquaintance, it is the most extensively technique that used in the higher order sliding mode methodology. However, in several cases, the chattering amplitude result from conventional method is less than the one result from super twisting algorithm. In this thesis, the describing function method, most suitable technique to analyze nonlinear systems, is used to make comparison between the two methods, the conventional and the super twisting sliding mode control. A simulation is performed to confirm the results that describing function method shows. Several situations illustrate the efficiency of first order sliding mode control over the higher order sliding mode control are represented.

Dedication

I wish to dedicate this thesis to my family:

My wife Fatma

My son Mohammed

My son Mahmud

My mother, my father, my brothers, my sisters, and my friends.

Acknowledgments

First and Foremost, I would like to thank Allah, the Almighty, for having given me courage, power, and the strength to pursue this degree.

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Chapter 1: Introduction

1.1 Sliding Mode Control (overview)

The notion of sliding mode control (SMC) appeared in the Former Soviet Union in the early 1960s. Indeed, in the mid 1970s the control committee system started to adapt the sliding mode theory outside Russia. In 1976 Itkis published a book about sliding mode theory then is followed by and survey paper by Utkin in 1977 [1]. Since then, several researches have been done in both theoretical and practical aspects of the sliding mode control [1] [2] [3]. As a result of to its order reduction feature and insensitivity to disturbances and variations of system parameters, the sliding mode control has been a principally appropriate technique for handling nonlinear systems with undetermined dynamics and disturbances. Furthermore, the sliding mode control may decrease the complication of feedback control design by means of decoupling the system into autonomous subsystems of lower dimension. Due to these properties, the sliding mode control theory has been applied to a wide range of problems, for example, space systems and robotics, chemical process, control of electric motors, automatic flight control, and helicopter stability augmentation systems [1].

The essential objective of sliding mode control is to impose the motion of the sliding mode in switching surfaces in the state space of the system using discontinuous control. The switching manifold or the discontinuity surfaces should be chosen such that sliding motion would show desired motion dynamics in stratification with specific performance condition.

There are, as stated before, two essential features in achieving this motion. First, enforcing sliding mode leads to a reduction of the system order. This reduction may result in the simplification and decoupling of design procedures. This advantage of sliding mode is significantly useful especially in the case of high-order nonlinear dynamic systems. The second main feature of sliding mode is the low sensitivity to disturbances and system parameters alteration. Consequently, sliding mode eradicates the requirement of precise modeling especially with nonlinear systems that run under parameter variation and uncertainty condition. Additional reason to consider sliding mode control is that discrete control behavior takes place naturally in several physical systems [2].

Consider the following affine system

$$\dot{x} = f(x,t) + B(x,t)u \quad (x, f \in \Re^{n}, B \in \Re^{n\times m}, u \in \Re^{m}). \tag{1.1}$$

Then, there are certain conditions where the control u is selected to compel the motion of the sliding mode among the intersections of m switching surfaces $s_1 = 0, s_2 = 0, \ldots, s_m = 0$ as follows.

$$u(x,t) = \begin{cases} u_i^{+}(x,t) & \text{if} \quad s_i(x) > 0 \\ u_i^{-}(x,t) & \text{if} \quad s_i(x) < 0 \end{cases} \quad (i = 1, \ldots, m) \tag{1.2}$$

where the elements of the vector f, $s_i(x)$, and the two scalar functions $u_i^+(x,t)$ and $u_i^-(x,t)$ are considered to represent continuous smooth functions. The existence condition of the sliding mode is equivalent to the condition of the stability of the motion in the subspaces $S(x) = [s_1(x), s_2(x), \dots, s_m(x)]^T$

$$\dot{s} = Gf(x) + GB(x)u \tag{1.3}$$

where $G = (\partial s/\partial x), (G \in \mathfrak{R}^{n\times m})$. The geometrical considerations of the existence condition of the sliding mode in plants with scalar discontinuous control functions can be found in [4]. The condition for sliding mode to exist requires the divergence from the switching surface s_i and $\dot{s}_i$, its time derivative, should have different signs in the neighborhood of a discontinuous surface $s_i(x)$=0 [4]; or

$$\lim_{s\to+} \dot{s}_i < 0 \quad \text{and} \quad \lim_{s\to-} \dot{s}_i > 0 \tag{1.4}$$

Under the condition $\dot{s}_i < -\alpha_i \, if \; s_i > 0, \dot{s}_i > \beta_i \, if \; s_i < 0, \alpha \; and \; \beta > 0$ the inequality (1.4) represents the reaching condition. It is the stipulation for the state to attain the surface $s_i(x) = 0$ from any arbitrary in initial finite time. Nevertheless, it is not required that each discontinuity surfaces satisfy the inequality (1.4) for the sliding mode to exist among the intersections of m switching surfaces $s_1 = 0, s_2 = 0, \dots, s_m = 0$ as in figure 1.1. Instead, the formulation of the existence conditions should be done in respect of stability of the origin in m−dimensional subspace $(s_1, s_2, \dots, s_m)$. That is, state trajectories must converge to the subspaces $S(x) = [s_1(x), s_2(x), \dots, s_m(x)]^T$ and reach these in a finite time. The importance of the finite time convergence is to distinguish between the continuous systems with asymptotically converging state trajectories to some manifold and the systems with sliding modes.

For example, the converging of the state trajectories of the system

$\ddot{x} - x = 0$ to the manifold $s = \dot{x} - x$ is asymptotic because $\dot{s} = -s$; yet, it would hardly be sensible to call the motion in $s = 0$ a "sliding mode" [5].

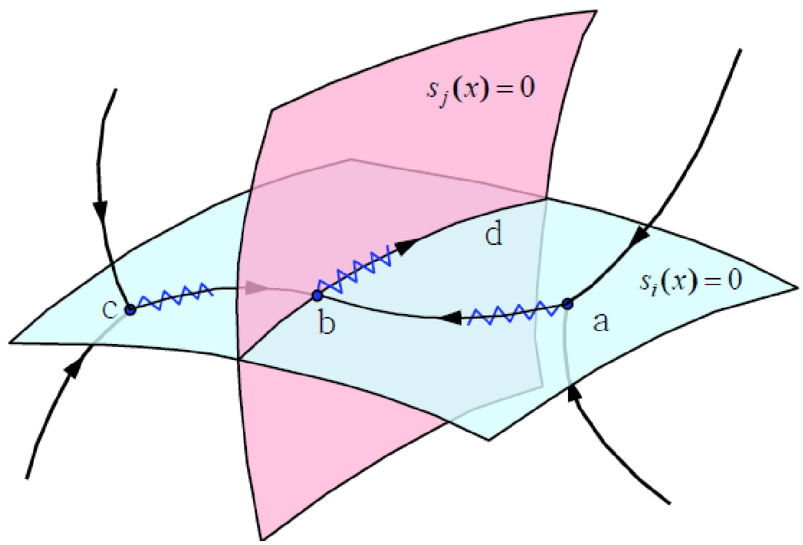

Figure 1.1: sliding mode along the intersection of multiple discontinuity surfaces.

When the states trajectories reach the sliding mode, the motion equation can be found applying the equivalent control technique [2] [6]. Let the matrix GB be a nonsingular matrix for any system state x, then the equivalent control $u_{eq}(x)$ is obtained as

$$\dot{s}(x) = Gf(x) + GB(x)u_{eq} = 0 \;\Rightarrow\; u_{eq} = -\big(GB(x)\big)^{-1}Gf(x) \tag{1.5}$$

Generally, the control action that is required to preserve an ideal sliding mode motion on $s(x)$ is called equivalent control [1]. Using a low-pass filter and filtering out the high-frequency component lead to derived the slow component of the real control which is, reasonably, assume to be equal the equivalent control $u_{eq}(x)$. Its time constant must be adequately small to maintain the slow component appropriate but considerable enough to remove the high-frequency component [7]. By substituting $u_{eq}(x)$ into the equation (1.1) the sliding mode equation can be found as

$$\dot{x} = f(x) - B(x)\big(GB(x)\big)^{-1}Gf(x) \tag{1.6}$$

After enforcing sliding mode along the manifold $S(x) = [s_1(x), s_2(x), \dots, s_m(x)]^T = 0$, precisely m of the states can be extracted as a linear combination of the residual $n - m$ states. Therefore, the sliding motion relies on the dynamics of $n - m$ states and hence a reduction in order turn up, yielding to simplification and decoupling of design procedures.

An essential characteristic of sliding mode is the advanced robustness against disturbance and insensitivity to parameter variations. To investigate this feature, take into consideration the next affine system

$$\dot{x} = f(x,t) + B(x,t)u + h(x,t) \quad (x, f \in \Re^n , B \in \Re^{n\times m} , u \in \Re^m). \tag{1.7}$$

The vector $h(x,t)$ represents the parameter variations and disturbances. Applying the equivalent control method gives the $u_{eq}(x)$ as follow

$$u_{eq}(x) = -\big(GB(x)\big)^{-1} G\big(f(x) + h(x)\big) \tag{1.8}$$

Substituting u_{eq} in equation (1.8) into equation (1.7) yields the next motion equation

$$\dot{x} = f(x) - B(x)\big(GB(x)\big)^{-1} Gf(x) + \Big(I_n - B(x)\big(GB(x)\big)^{-1} G\Big) \tag{1.9}$$

Where the $range$ of $B(x)$ is formed by the base vectors of $B(x)$ for every point (x,t). Then the sliding mode is $h - invariant$ if

$$h(x) \in range\ B(x) \tag{1.10}$$

The condition (1.10) represents the matching condition [7] [8], implies that there exists a vector $\varphi(x,t)$ such that

$$h(x,t) = B(x,t)\ \varphi(x,t) \tag{1.11}$$

Substituting equation (1.11) back in equation (1.7) shows the sliding motion in any manifold $s = 0$ does not depend on the vector $h(x,t)$. The upper limit of $h(x,t)$ is only needed to ensure the existence condition (1.4). This reason leads the sliding mode to be a preferred option when dealing with high-order systems operating under uncertain condition [5].

1.2 The Chattering Problem

In real systems, there exist an ignored dynamics from imperfect sensors or processors. Such unmodeled dynamics are the main problem of the implementation of sliding mode control theory, since they lead to an inadmissible occurrence of oscillation having amplitude finite frequency and, this phenomenon is known as 'chattering'. This phenomenon is disadvantageous since it leads to unacceptable control accuracy. If there are neglected rapid dynamics in the perfect model, the chattering may emerge since an idyllic sliding mode may not occur

1.2.1 Causes of Chattering

The term "chattering" refers to the existence of finite amplitude, finite-frequency oscillations occurring in several sliding mode applications due to the neglected dynamics when modeling the plant or the imperfections of switching apparatus of the system. Two essential causes of chattering have been determined in the literature. The first happens in implementation of sliding mode control in discrete-time, e.g. with a digital microcontroller. In this case, the Chattering happens because the limited switching frequency due to the sampling rate, but accurate implementation of sliding mode control presupposes infinite switching frequency. Indeed, this sort of chattering is not a problem when dealing with a system that applies discontinuous control law directly to analog device. The second type is due to the rapid neglected dynamics which get energized by sliding mode controllers have high frequency switching leading to unacceptable oscillations. More details about the first type can be found in [2].

Unmodeled dynamics come from actuators and sensors are usually ignored when designing the controller. For plants with sliding mode controllers, rapid switching can excite these unmodeled dynamics leading to chattering phenomena.

As stated in singular perturbation theory [9] [10], for continuous time system, rapid motion elements with large bandwidth fade fast resulted stability of these components, for this reason the actuator's dynamics may be neglected. However, in plants with discontinuities, the solution to the motion equation relies on the small time constants of rapid elements. Contrasting to plants with continuous control, discontinuities in the control action provoke the unmodeled dynamics leading to oscillations in the chattering behavior. Therefore, unmodeled dynamics cannot be ignored and should be accounted when designing controllers for plants with discontinuities.

1.2.2 Solutions for Chattering Problem

Improper treatment of Chattering in the control design has been a main problem for many real life applications when implementing sliding mode.

Fortunately, avoiding chattering does not presuppose a comprehensive model of all plant's components. Alternatively, sliding mode controller should be designed such that the entire preferred characteristics are achieved under the assumptions that there are no unmodeled dynamics. Then, potential chattering is to be minimized using some efficient methods. Next, a brief review of most remarkable and acknowledged solutions to the problem of chattering are presented.

1.2.2.1 Boundary Layer Solution

The use of Boundary layer based sliding mode control to decrease chattering was first offered by Slotine and Sastry [11] and Soltine [12]. The base of this method is to lessen the stringent condition of "movement on sliding surface" to achieve approximately sliding mode or Quasi Sliding Mode. This is obtained by replacing the discontinuous function $\mathrm{sign}(s)$ in a vicinity layer of the manifold $s(t) = 0$ by an approximating continuous saturation function $\mathrm{sat}(s)$ leading to avoid the discontinuities of the control and switching control action . The major problem of the boundary layer method is that a real sliding mode may not occur when the discontinuous switching function is replaced by an approximation continuous switching function. Therefore, it is not guaranteed that the trajectories inside the vicinity of sliding mode converge to zero. Thus, the accurateness and sturdiness of the sliding mode are partially omitted by this technique.

1.2.2.2 Observer-Based Solution

Presented by Bondarev, Kostyleva, and Utkin [14], the observer-based solution is assembled on the applying an auxiliary observer loop instead of the main control loop to produce an ideal sliding mode control without chattering. Using software, an ideal sliding mode control can entirely generated and although a discontinuous control function is applying as an input signal to the actuator, the plant performs as if an equivalent control is implemented on the system. A chattering-free state trajectory is the result of following the main control loop to the observer loop.

1.2.2.3 State-Dependent Gain Solution

The first appear of the state-dependent gain solution was in the publications on variable structure control in 1960s [15]. It supplies a technique to decrease chattering without use an additional system. The notion is stand on the observation that the chattering magnitude is relative to the gain of the discontinuous control M. Hence, decreasing M such that sliding mode existence condition is not broken will decrease chattering. Consider the following sliding mode controller in the second order system [16]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ x_2 &= a_1 x_1 + a_2 x_2 + u \end{aligned} \tag{1.12}$$

$$s = c x_1 + x_2 \tag{1.13}$$

$$u = -Msign(s) = \begin{cases} -M \;\; if \;\; s > 0 \\ +M \;\; if \;\; s > 0 \end{cases} \tag{1.14}$$

Instead of being constant, the gain M is selected to be a function of the state

$$M = M(x_1) = M_0(|x_1| + \xi) \tag{1.15}$$

where c and M_0 and ξ are positive constants, but ξ is a adequately small. The constant M_0 must be chosen such that sliding mode is permanently enforced along the switching surface $s = 0$. Hence, sliding mode is enforced if

$$M_0 \geq |a_1 - a_2 c - c^2| \tag{1.16}$$

By using simulation, it can be demonstrated that chattering is decreased in the existence of unmodeled dynamics by using the state-dependent gain method [17].

1.2.2.4 Equivalent Control-Dependent Gain Solution

The chattering magnitude is proportional to the amplitude of the discontinuous control gain M. Thus, reducing M (while still preserving sliding mode enforcement condition) leads to decreasing chattering amplitude. State dependent gain methodology uses this notion. However, this method is barely valid in nonlinear plants with unknown disturbances. The average value of the discontinuous control u is the equivalent control u_{eq}. Hence, applying the discontinuous control u to a through a low pass filter, it generates the equivalent control u_{eq}, which is used to regulate the amplitude of the discontinuous control u such that sliding mode is preserved once it has enforced [16] [17].

Chapter 2: Higher Order Sliding Mode

2.1 Introduction

The key difference between the higher order and conventional and sliding mode is that instead of using first order derivatives of the sliding variable (s) the higher order derivatives are used [18] [19] [20]. The unsophisticated technique to resist uncertainty is to preserve several constraints through brute force. For example, move back any divergence of the system to the constraint by an adequately robust effort. This method guides directly to first-order sliding modes.

As said before, beside its low sensitivity to disturbances and variations of system parameters, the first-order sliding modes has a very serious drawback which is chattering phenomena. Although, several methods were presented to handle this problem [16] [20], the significant accuracy and robustness of the sliding mode were missed to a certain degree. Instead of modifying the first deviation of the sliding variable (s) as in the condition of first-order sliding modes, the higher order-sliding mode is based on operating the higher order-time-derivatives of the plant deviations from the boundary constraint [21]. As in the case of the first order sliding modes, HOSM has the advantage of robustness. Moreover, it has an additional advantage of chattering attenuation or even sometimes removal.

Due to its chattering reduction property and successful implementation, the interest of control community withdrew towards this methodology and several algorithms have been introduced. The sliding variables and its derivatives decide the sliding order. If $s = 0$ is forced then it is called first order sliding mode, the same way if $s = \dot{s} = 0$ is forced then it becomes the case of the second order sliding mode and so on. Even though the 3^{rd} and higher order sliding mode approaches are used in industry, mostly 2^{nd}order is used. Next, notable algorithms 2^{nd} of order sliding mode [22] are presented.

2.2 Second Order Sliding Mode Algorithms

2.2.1 Twisting Algorithm

The twisting algorithm is an interesting class of the second order sliding mode control, and it was presented under the assumption that after a finite time interval the point $s = \dot{s} = 0$ will be reached. The control for this algorithm is a continuous function, and it is an output of an additional integrator with continuous input, that is

$$\dot{u} = v \tag{2.1}$$

$$\dot{v} = -M_o sign(s) - M_1 sign(\dot{s}) \tag{2.2}$$

where M_o and M_1 are positive constant or state functions, and the second derivative of the variable s is given by [2]

$$\ddot{s} = F(x, t, u) - M_o sign(s) - M_1 sign(\dot{s}) \tag{2.3}$$

The function $F(x, t, u)$ is a continuous function, where $F(x, t, u) \leq F_o$, where F_o is a constant value, and $M_o > M_1 + F_o$, $M_1 > F_o$. The input v undergoes discontinuities on $\dot{s} = 0$, but sliding mode does not exist on this switching line, if $s \neq 0$ because $M_o > M_1$ and $\ddot{s}$ does not change sign [7].

Since $\dot{s}$ is need for the implementation, that relative degree of this method is equal one.

The sufficient conditions for system (2.3) to sustain asymptotic stability is given by

$$M_o > M_1 + F_o, M_1 > F_o \tag{2.4}$$

Indeed, the first inequality means that function $\dot{s}$ can not be equal to zero identically if $s \neq 0$ [7].

This algorithm twists around the origin as shown in Fig. 2.1. The trajectories perform an infinite number of rotations and converge to the origin in finite time. The magnitude vibration amplitude and rotation time reduce in geometric evolution [21].

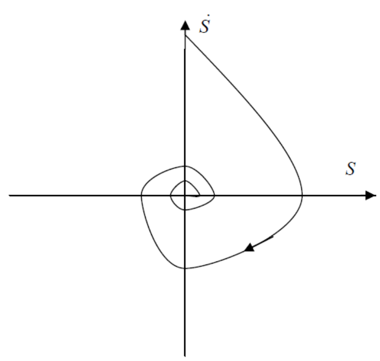

Figure 2.1. Phase trajectory of Twisting algorithm

2.2.2 Sub-Optimal Algorithm

Sub-optimal algorithm was engineered as a sub-optimal feedback realization of a conventional time optimal control for a system with double integrator [21]. Making the relative degree equal to 2.

The trajectories of this algorithm are restricted within bounded parabolic arcs that contain the origin. The trajectory of the state possesses two behaviors, twisting and skipping, as shown in Fig. 2.2. Here the system equation and control law are defined as [21] [24]

$$\ddot{x} = u \tag{2.5}$$

$$u(t) = \begin{cases} -M_1 & if \quad \left[s(t) - \frac{s^*}{2}\right] > 0 \\ M_2 & if \quad \left[s(t) - \frac{s^*}{2}\right] \le 0 \end{cases} \tag{2.6}$$

where M_1, M_2 are positive constant, and s^* is the value of s at the last time when $\dot{s}(t) = 0$. The second derivative of s is given by

$$\ddot{s}(t) = u(t) + F(t), |F(t)| < F_o = constant \tag{2.7}$$

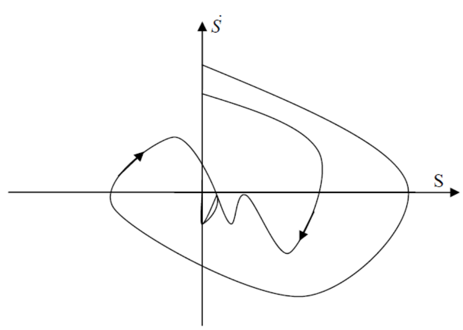

Figure 2.2. Phase trajectories of Sub-optimal algorithm

For initial conditions $x(0) > 0$, $\dot{x}(0) = 0$ the optimal process includes two intervals. The state x decays from $x(0)$ to $x(0)/2$ at the first interval with $u = -M$ and then from $x(0)/2$ to 0 at the second interval with $u = M$ [23].

2.2.3 Super Twisting Algorithm

Super twisting algorithm is used to overcome chattering problem in variable structure systems for the system with relative degree one. To formulate the control law $u(t)$, two terms are combined together; one is a discontinuous time derivative function, and the second is the continuous time function of the sliding variable. The corresponding phase portrait of super twisting algorithm trajectories is shown in Fig.2.3.

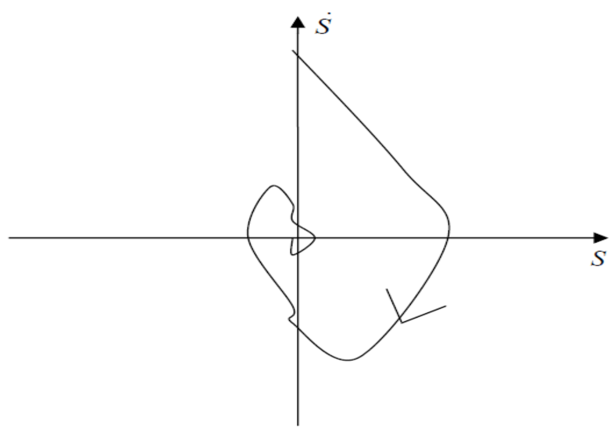

Figure 2.3. Phase trajectory of Super-twisting algorithm

Similar to the twisting algorithm, the super twisting sliding mode control algorithm implies inserting an integrator into the control loop. However, the relative degree of the system is increased, since, in contrast to the twisting algorithm, the time derivative of function s is not needed. The super twisting sliding mode control algorithm has advantage to enforce the second order sliding mode after a finite time and repressing unknown disturbances with bounded time derivatives [7].

The continuous control for the super twisting algorithm is given by [30] as follow

$$u = -a\sqrt{|s|}\, sign(s) + v \tag{2.8}$$

and $\dot{v}$ is given by

$$\dot{v} = -M\, sign(s) \tag{2.9}$$

The parameters a and M are positive and constant, that is, $a > 0, M > 0$

The second time derivative of function s is

$$\ddot{s} = -a\frac{\dot{s}}{2\sqrt{|s|}} - M\, sign(s) + \dot{f}(t) \tag{2.10}$$

Where $f(t)$ is a function of the disturbances and state, that is, $|f| \le f_o$, and f_o is constant.

The important feature of super twisting algorithm is that the information about time derivatives of the sliding variable and system parameters are not required. Hence, the observer can be omitted and the complexity of the design is reduced.

Chapter 3: Describing Function Method (overview)

3.1Introduction

A nonlinear system is a dynamic system that does not follow the principle of superposition. In real life, all dynamic systems are nonlinear to some extent. Linear analysis techniques are typically used for a system that does not possess a significant nonlinear behavior. On the other hand, linear system analysis methods generally fail if a system has harsh nonlinear characteristics and is based on a broad range of operating conditions. a number of nonlinear system analysis approaches such as Lyapunov's first and second methods, the phase plane technique, and the describing function methods are regularly used to analyze the control of dynamic systems that have sever nonlinearity. In General, there is no universally valid analysis technique for nonlinear systems; each method is specific to a particular set of problems. This chapter explains different types of the describing function method and their capabilities for nonlinear system analysis.

3.2 Types

The frequency response approach is an extremely helpful technique for the analysis of linear systems. For nonlinear systems, an approximate frequency response approach, the describing function method, is widely used and very useful. The principle idea of the describing function is to substitute the system nonlinearity by an optimum quasi-linear approximator. The optimum quasi-linear approximator is a function that has a sinusoidal input characteristics, amplitude and frequency [25]. Therefore, a frequency domain representation of the nonlinear system can be formulated using describing functions. The describing function is achieved by assuming a specific structure of input to the nonlinearity, then computing the "equivalent gain" of the nonlinearity relevant to the presumed input. The most general structures of an assumed input are bias inputs, sinusoidal inputs and random inputs among a Gaussian distribution. Several types of describing functions can be extracted depending on the different sets of inputs adopted. If the nonlinearity is adequately simple, for example, a simple component with hysteresis, analytical terms for the describing function can be obtained from tables offered by several authors [25] [26]. Describing function methods are valuable to investigate stability of nonlinear systems. The describing function method provides considerable understanding of several phenomena that nonlinear control system has, such as jump resonance and limit cycles. In addition, the describing function method allows to find the stability margin for several signal levels of a nonlinear system in the course of an amplitude-dependent gain margin [26]. The most advantage of the describing function approach is its capability to deal with complex nonlinear systems or problems. For example, the describing function method can be used to analyze higher order systems without much difficulty.

The main restriction of the describing function method is that the structure of the input signal to the nonlinearity should be assumed in beforehand. Hence, the conclusion made by this method is valid for this certain situation only. Any conflict between the assumed input and the real input to the nonlinearity inserts an extra approximation to the describing function method. Another restriction of this method is that there is no pleasing evaluation of the precision of the method. Yet, experiment in applying the describing function method confirms that this quasi-linear approximation is widely correct for several nonlinear systems [26].

3.2.1 Single-sinusoidal Input Describing Function (SIDF)

Between all the describing function methods, the one that is mostly used is the single-sinusoid input describing function (SIDF). It is obviously clear from its name; the input signal to the nonlinearity is considered a single sinusoid. The SIDF is defined to be the complex ratio of the fundamental component of the nonlinear element by the input sinusoid[27].

Consider the next function as an input to the nonlinearity

$$u(t) = A \sin(\omega t) \tag{3.1}$$

where A and ω are the amplitude and frequency of the input signal respectively. The SIDF is denoted by $N(A, jw)$ and can be written as

$$N(A, jw) = \frac{A_1}{A} e^{j\phi_1} \tag{3.2}$$

where A_1 is the fundamental component amplitude of the output function, and ω is the difference in the phase between the fundamental component of the input the and output . Here, both A_1 and ϕ are functions of the amplitude A and frequency ω of the input signal. In the case of static nonlinearities, the output of nonlinearity relies only on the immediate value of the input function, and the time derivative is not involved. Hence, the describing function does not depend on the input frequency . Furthermore, the describing function is said to be real valued function if the nonlinearity is static and single-valued [28].

Theoretically, The SIDF can be found analytically in any given nonlinearity. However, practically, nonlinear characteristics are usually either extremely complex to be described analytically, or they are only can be represented by measuring experimentally the physical system with no available analytical relationships. Several approximate techniques are accessible to compute the describing function for these conditions [25]. The numerical evaluation of integrals is the method of interest here to calculate the describing function., if the input function to the nonlinearity in a dynamic system is given by the equation (3.1), then the SIDF can be represented as

$$N(A, j\omega) = \frac{2}{\pi A} \int_{\omega t=0}^{\omega t=\pi} y(A \sin \omega t) \sin(\omega t)\, d(\omega t)$$

$$+ j \frac{2}{\pi A} \int_{\omega t=0}^{\omega t=\pi} y(A \sin \omega t) \cos(\omega t)\, d(\omega t)$$

$$= \frac{2}{\pi A} \int_{\omega t=0}^{\omega t=\pi} y(A \sin \omega t) d(-\cos \omega t) + j \frac{2}{\pi A} \int_{\omega t=0}^{\omega t=\pi} y(A \sin \omega t) \cos(\omega t)\, d(\omega t) \qquad (3.3)$$

where y is the output of the nonlinearity.

Considering any type of nonlinearity or any systems with several nonlinearities, and assume the input to the nonlinear system is

$$u(t) = A\cos(wt) \tag{3.4}$$

Then the describing function method can be written as

$$N(A, j\omega) = \frac{\omega}{\pi A}\int_{(k-1)T}^{kt} y(t)e^{-j\omega t}dt \tag{3.5}$$

where k is an integer and indicates the k^{th} period of the output. For the system to achieve steady state, the value of k should be sufficiently large. Here the equation (3.5) is equivalent to equation (3.3). This formula can be extended to describe higher harmonics as following

$$N_m(A, j\omega) = \frac{\omega}{\pi A}\int_{(k-1)T}^{kt} y(t)e^{-jm\omega t}dt \tag{3.6}$$

where m is an integer indicates the m^{th} harmonic response [28].

Contrasting with linear systems, since the superposition rule does not hold in nonlinear systems the frequency response data characterized by describing functions cannot be reversed to find the precise time response to the given input,. However, for nonlinear systems possess weak nonlinear behavior, the SIDF still provides valuable approximate results [25]. It comes out to be an issue in the frequency response analysis when the describing function being a function of both the amplitude A and the frequency ω.

Consider the closed loop system in Figure 3.1, where $G(s)$ and $N(a)$ represent the transfer function of the linear part and the nonlinear part of the system respectively.

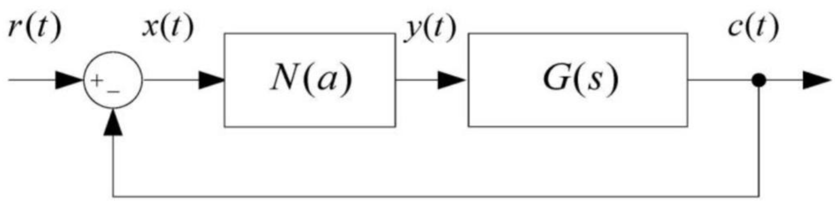

Figure 3.1 General nonlinear control system

The functions $r(t)$ and $x(t)$ are assumed to be single sinusoid functions, that is,

$$r(t) = M \sin(wt) \tag{3.7}$$

$$x(t) = A \sin(wt) \tag{3.8}$$

where M and A are the amplitude of $r(t)$ and $x(t)$ respectively. Applying the SIDF method, The closed loop transfer function is given as

$$\frac{C}{R}(j\omega, A) = \frac{N(A,j\omega)G(j\omega)}{1+N(A,j\omega)G(j\omega)} \tag{3.9}$$

where $N(A, j\omega)$ is the Single-sinusoid Input Describing Function of the nonlinear part of the system, and the sensitivity function is given by

$$\frac{X}{R}(j\omega, A) = \frac{1}{1+N(A,j\omega)G(j\omega)} \tag{3.10}$$

From the equation (3.8) and (3.9), these two functions are functions of A and ω. Since the value of A is not available, the suitable option of $N(A, j\omega)$ is also not available.

The following procedure is suggested to resolve this circular problem. First, the value of A must be assumed at any specified input frequency ω and then the describing function $N(A, j\omega)$ can be found. The value of M corresponding to A while working with the magnitude of $\frac{X}{R}$ is specified as

$$M = \frac{A}{\left|\frac{X}{R}(j\omega,A)\right|} \tag{3.11}$$

By inspecting the accumulated data derived above the value of A related to a particular combination of M and ω can then be found. In this case, the transfer function $\frac{C}{R}$ may be calculated, hence determining the closed loop frequency response of the system [25].

3.2.2 Two-sinusoid Input Describing Function (TSIDF)

There are situations where the input to the nonlinear part of a system is periodic but not sinusoidal, hence applying the SIDF method is not valid. There are cases where the input is approximated by two sinusoids combined together. Under this condition, the two-sinusoid input describing function is applied. Here, the input signal to the nonlinearity is considered to be

$$x(t) = A\, sin(\omega_A t + \theta_A) \; + \; B\, sin(\omega_B t + \theta_B) \tag{3.12}$$

In general, the Two-sinusoid Input Describing Function is a function of six parameters, the amplitudes A and B, the frequency ω_A and ω_B, and the phase angles θ_A and θ_B. In some cases one input sinusoid signal is related to the any harmonic component of the other input signal. In such a situation the TSIDF depends on the relative phase angle $(\theta_A - \theta_B)$ instead of the two phase angles independently [25]. The TSIDFs can be given as

$$N_A = \frac{phasor\ representation\ of\ output\ component\ of\ frequency\ \omega_A}{phasor\ representation\ of\ input\ component\ of\ frequency\ \omega_A} \tag{3.13}$$

$$N_B = \frac{phasor\ representation\ of\ output\ component\ of\ frequency\ \omega_B}{phasor\ representation\ of\ input\ component\ of\ frequency\ \omega_B} \tag{3.14}$$

where the subscript linked to N denotes the input component to which the describing function applies.

Many methods for computing the TSIDFs analytically, for example, double Fourier series expansion, direct expansion, and power series expansion are represented in the [25]. The TSIDF technique is applicable to determine the regions of stable operation, harmonic content, and sub-harmonic oscillation and [29]. The TSIDF can also be used to study the complete forced harmonic response of a limit cycling system and, similarly, the response of a non-limit-cycling system to two simultaneously applied sinusoids.

3.2.3 Random Input Describing Function (RIDF)

The random input describing function (RIDF) is another used type of the describing function is. In the case of (RIDF), the input to the nonlinear part is assumed to be a random signal, and it does not have a characteristic wave form. Here, it is essential that a statistical approach is required and no another deterministic interpretation is applied. numerous technique s to calculate the Random Input Describing Function and tables of the RIDF for some uncomplicated nonlinearities are available in the literature [25].

Generally speaking, the basic idea of the describing function method is based on the four following assumptions

1. The system has only single nonlinear component. This implies that if there are several nonlinearities in a system, they should be lumped together as a single nonlinearity, or only the primary nonlinearity is considered.

2. The nonlinear component has time-invariant characteristic. The second assumption implies that only autonomous nonlinear systems are considered.

3. Corresponding to a sinusoidal input $= sin(\omega t)$, only the fundamental component of the output has to be considered. This assumption implies that the all the harmonics except the fundamental component are neglected in the analysis. For this assumption to be valid, the linear element following the nonlinearity must have low-pass properties.

4. The nonlinearity is odd. This implies that the relation between the input and output of the nonlinear component is symmetric about the origin.

The above assumptions have been broadly studied in literature, leading to describing function methods for general situations, such as time-varying nonlinearities, multiple nonlinearities, or multiple-sinusoids [27].

Chapter 4: analysis of conventional and super twisting sliding mode control

4.1Introduction

In several literatures and application, the super twisting algorithm is suggested as a solution of chattering problem that appears the first order (conventional) sliding mode controller in the presence of an unmodeled dynamic. To achieve better understating and make a clear decision about that suggestion, a comparison between the two techniques is done in this chapter using analytical technique, the describing function method, and Matlab simulation. Two main conditions is considered, first the disturbance that may interfere with the system is neglected $F = 0$. The second situation is the most practical situation in which the disturbance does not equal zero, $F \neq 0$.

4.2 Part one: The disturbance $\boldsymbol{F = 0}$

4.2.1 Analysis of conventional sliding mode control

First, the analysis is applied to the system with conventional sliding mode controller; the block diagram for the plant is shown in figure 4.1.

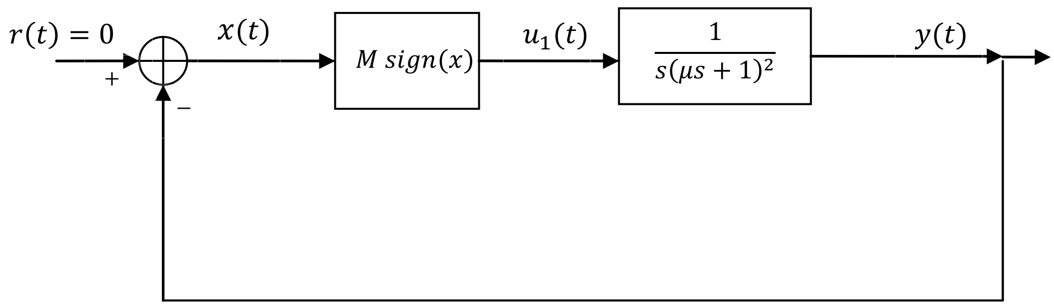

Figure 4.1 Simplified representation of CSMC scheme

The function $x(t)$ is considered to be $A\sin(\omega t)$ and the output of the nonlinearity is approximated to be

$$u_1(t) = \frac{a_o}{2} + a_1\cos(\omega t) + b_1\sin(\omega t) \tag{4.1}$$

Because the odd symmetry of the relay function, one has

$$\begin{aligned} a_o &= 0 \\ a_1 &= 0 \end{aligned} \tag{4.2}$$

Then the $u_1(t)$ becomes

$$u_1(t) = b_1\sin(\omega t) \tag{4.3}$$

Where b_1 is given by

$$b_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} M\sin(\omega t)dt \tag{4.4}$$

Computing the integration yields

$$b_1 = \frac{4M}{\pi} \tag{4.5}$$

The describing function is frequency independent and it is given by

$$N(A) = \frac{u_1(t)}{A} = \frac{4M}{\pi A} \tag{4.6}$$

The transfer function of the linear part is given by

$$G(s) = \frac{1}{s(\mu s+1)^2} \tag{4.7}$$

Replacing the Laplace parameter s by $j\omega$ yields

$$G(j\omega) = \frac{1}{j\omega(\mu j\omega+1)^2} \tag{4.8}$$

The inverse transfer function of the linear part is needed to complete the analysis, and it is given by

$$\mathcal{g}(j\omega) = (2\mu\omega^2) + j(\mu^2\omega^3 - \omega) \tag{4.9}$$

Equating $N(A)$ to $\mathcal{g}(j\omega)$ yields

$$\frac{4M}{\pi A} = (2\mu\omega^2) + j(\mu^2\omega^3 - \omega) \tag{4.10}$$

Here, the imaginary part of $N(A)$ is equal zero, that is,

$$(\mu^2\omega^3 - \omega) = 0 \tag{4.11}$$

$$\Longrightarrow$$

$$4M/_{\pi A} = (2\mu\omega^2) \tag{4.12}$$

Solving for ω and A in equation (4.10) yields

$$\omega = \frac{1}{\mu}$$
$$A = \frac{2\mu M}{\pi} \tag{4.13}$$

It is noted from the equation (4.13) as μ tends to zero the frequency ω and the amplitude A tend to infinity and zero respectively, figures 4.2 shows this behavior.

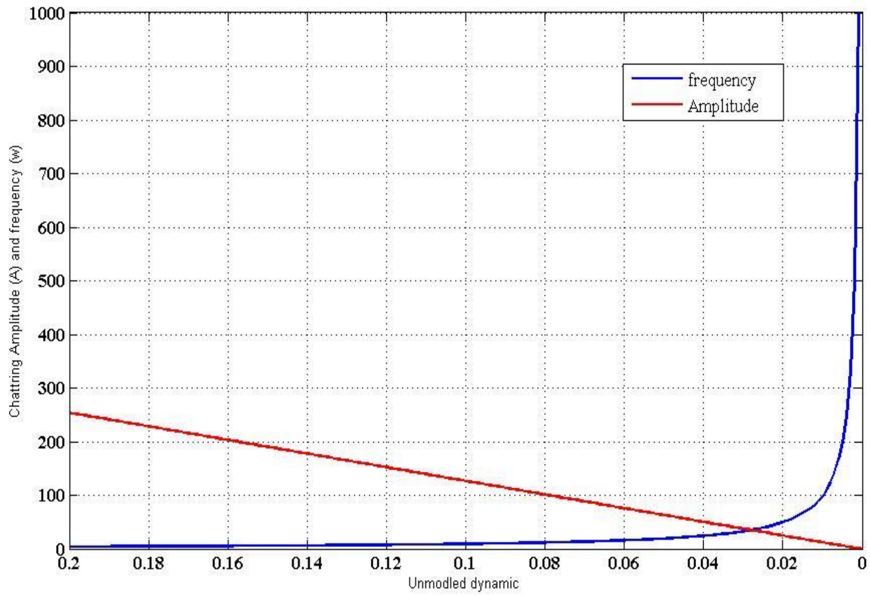

Figure 4.2 Chattering amplitude (A) and frequency (w) VS the unmodeled dynamic μ for CSMC

4.2.2 Analysis of super twisting sliding mode control

Here, the analysis is applied to the system with super twisting sliding mode controller; the block diagram for the plant is shown in figure 4.3.

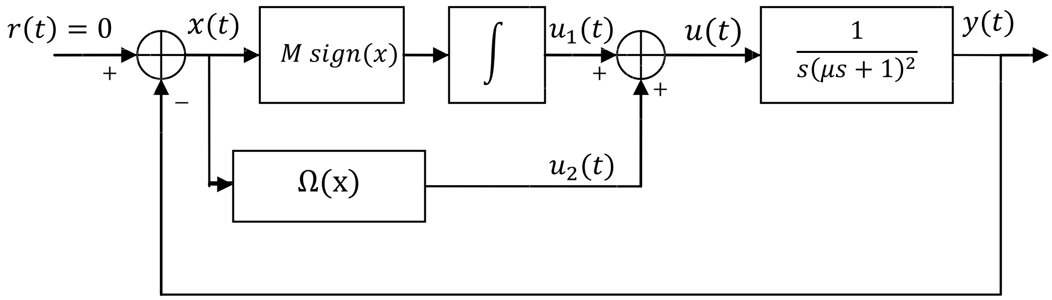

Figure 4.3 Simplified representation of STSMC scheme

The function of $\Omega(\mathrm{x})$ is given by

$$\Omega(\mathrm{x}) = a\sqrt{|x|}sign(x) \tag{4.14}$$

$$\Omega(\mathrm{x}) = a\sqrt{|Asin\ (\omega t)|}sign\big(Asin\ (\omega t)\big) \tag{4.15}$$

Since the function $\Omega(\mathrm{x})$is odd and symmetric, one has the describing function of this block as follow

$$N(A,\omega) = \frac{b_1}{A} \tag{4.16}$$

where $b_1$is calculated as

$$b_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} a\sqrt{|Asin\ (\delta)|}sign(A\sin\delta)sin\delta\ d\delta$$

$$b_1 = \frac{2a}{\pi}\int_0^{\pi}\sqrt{|Asin\ (\delta)|}sign(A\sin\delta)sin\delta\ d\delta \tag{4.17}$$

Since

$$sign(A\sin\delta)sin\delta = |sin\delta| \tag{4.18}$$

and the period is from 0 to π one has

$$sign(A\sin\delta)sin\delta = sin\delta \tag{4.19}$$

then b_1 becomes

$$b_1 = \frac{2a\sqrt{A}}{\pi}\int_0^{\pi}\sqrt{|sin\ (\delta)|}sin\delta\ d\delta \tag{4.20}$$

Due to uncertainty about the integral, an accepted estimation has been done by replacing $\sqrt{|sin\ (\delta)|}$ by $sin\delta$. it leads b_1 to be as next

$$b_1 = \frac{2a\sqrt{A}}{\pi}\int_0^{\pi} sin\delta\ sin\delta\ d\delta \tag{4.21}$$

where the absolute value of the sin function has been disregarded due to the positivity of the period. Computing the integral yields

$$b_1 = a\sqrt{A} \tag{4.22}$$

To obtain the describing function of $\Omega(x)$ b_1 is divided by A, that is,

$$N(A,\omega) = {}^{a}/_{\sqrt{A}} \tag{4.23}$$

Because the function is frequency independent, the previous equation can be written as

$$N(A) = {}^{a}/_{\sqrt{A}} \tag{4.24}$$

Combining the describing function of the relay multiplied by the integral and $\Omega(x)$ yields

$$N_t(A,\omega) = {}^{4M}/_{\pi A s} + {}^{a}/_{\sqrt{A}} \tag{4.25}$$

The block diagram of the system becomes

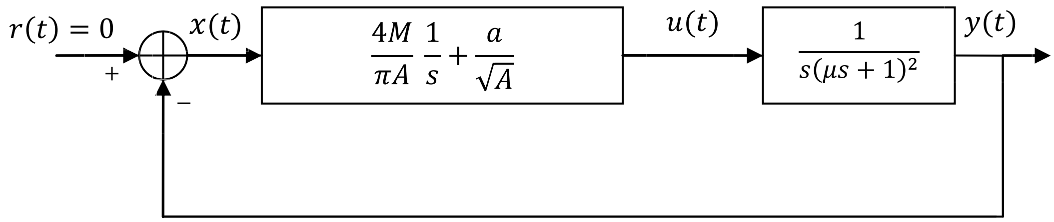

Figure 4.4 Simplified representation of STSMC scheme after applying the describing function.

Equating the equation (4.25) to the inverse transfer function of the linear part $g(j\omega)$, then solving for ω and A yields

$$A = \frac{a^2}{4\,\mu^2\,\omega^4} \tag{4.26}$$

$$\omega_1 = \frac{a\,\pi\left(a^2 + \frac{16\,M}{\pi}\right)^{\frac{1}{2}}}{\mu\,(\pi\,a^2 + 16\,M)} \tag{4.27}$$

$$\omega_2 = -\frac{a\,\pi\left(a^2 + \frac{16\,M}{\pi}\right)^{\frac{1}{2}}}{\mu\,(\pi\,a^2 + 16\,M)} \tag{4.28}$$

$$\omega_3 = 0 \tag{4.29}$$

Only ω_1 is the interesting root for ω. Substituting this root back in the equation (4.26) yields

$$A = \frac{\mu^2\,(\pi\,a^2 + 16\,M)^2}{4\,a^2\,\pi^2} \tag{4.30}$$

It is noted from the equation (4.30) as μ tends to zero the frequency ω and the amplitude A tend to infinity and zero respectively, figure 4.4 shows this behavior.

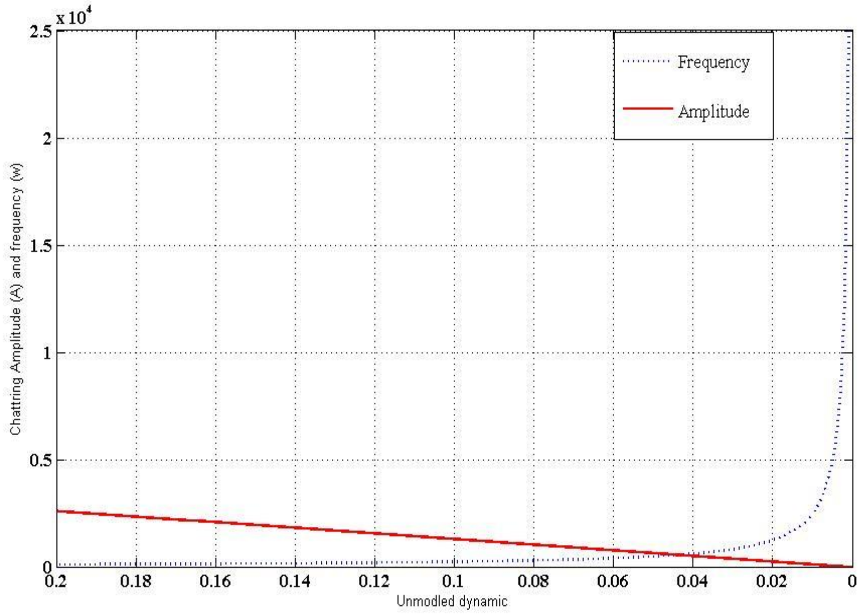

Figure 4.5 Chattering amplitude (A) and frequency (w) VS the unmodeled dynamic μ for STSMC

Next, the analysis is done considering two possible situation, $0 < \mu \leq 0.1$ and $\mu = 0.2$.

Case 1: 0 $< \mu \leq 0.1$

The plot in figure 4.6 shows the amplitude of chattering in both cases, conventional and super twisting controllers, versus the parameters a which is needed the guarantee the stability the system with super twisting controller. It is clear that the chattering amplitude of super twisting algorithm starts from big value then goes down until reach the a range in which its value is less than chattering amplitude of the conventional case. However, it can be observed that after that range, the chattering amplitude of super twisting algorithm grows so fast, and it is extremely bigger than chattering amplitude of the conventional controller. In addition, the figure 4.7, which is the result of the Matlab simulation of the block in figure 4.8, confirms the result of the analysis of the describing function method. Although the parameter a could be chosen to ensure that chattering amplitude of super twisting is always lees then chattering amplitude of the conventional one, this adjustment must be done very carefully since in some range the parameter a can make the chattering amplitude excessively high leading to destruction of the system.

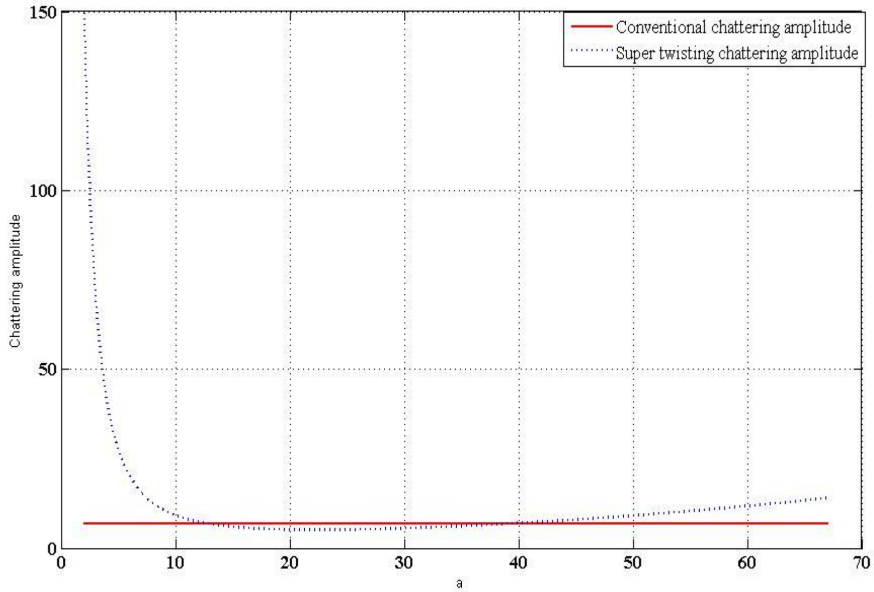

Figure 4.6 Chattering amplitude of CSMC and STSMC using DFM, $\mu = 0.1$

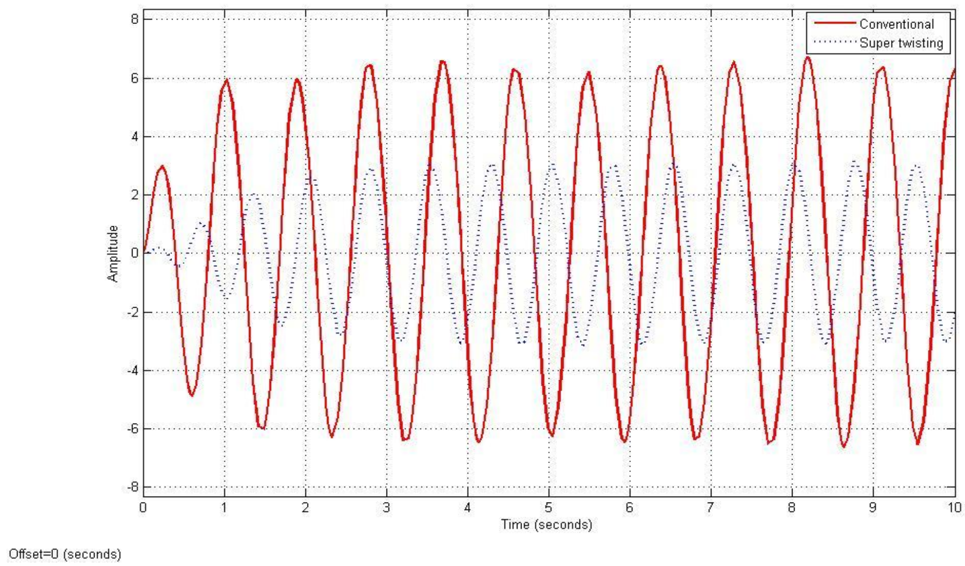

Figure 4.7 Chattering amplitude of CSMC and STSMC using simulation, $\mu = 0.1$

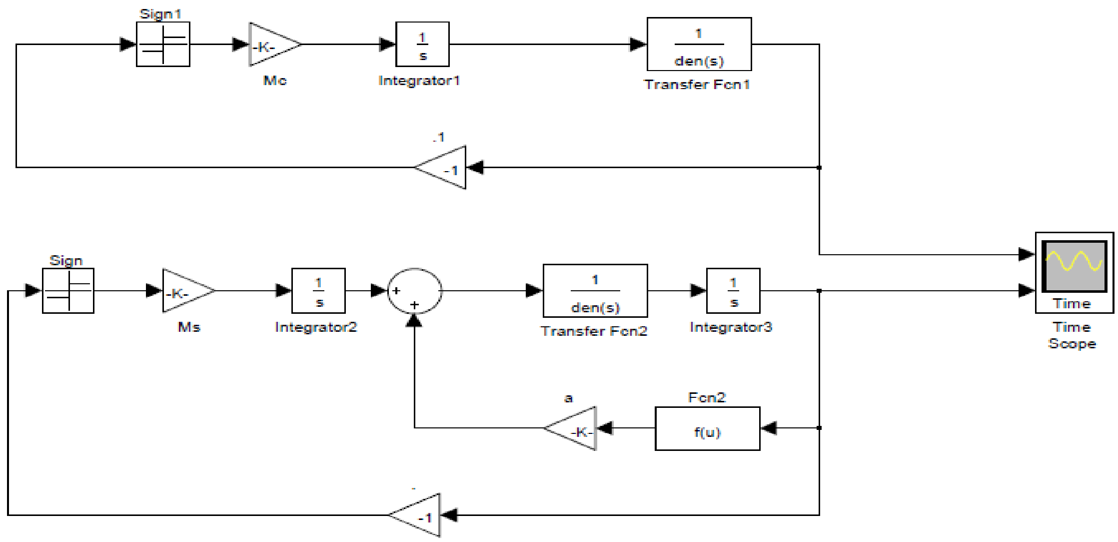

Figure 4.8 Matlab block simulation of CSMC and STSMC

Case 2: $\mu = 0.2$

Figure 4.9 and 4.10, which is the result of the Matlab simulation of the block in figure 4.8 at $\mu = 0.2$, show that, when $\mu = 0.2$, any adjustment for the parameter a can not make the chattering amplitude of super twisting algorithm lees then chattering amplitude of the conventional one. Therefore, the unmodeled dynamic effect the decision that any technique should be used. Hence, the claim that super twisting algorithm is not always better than the conventional on.

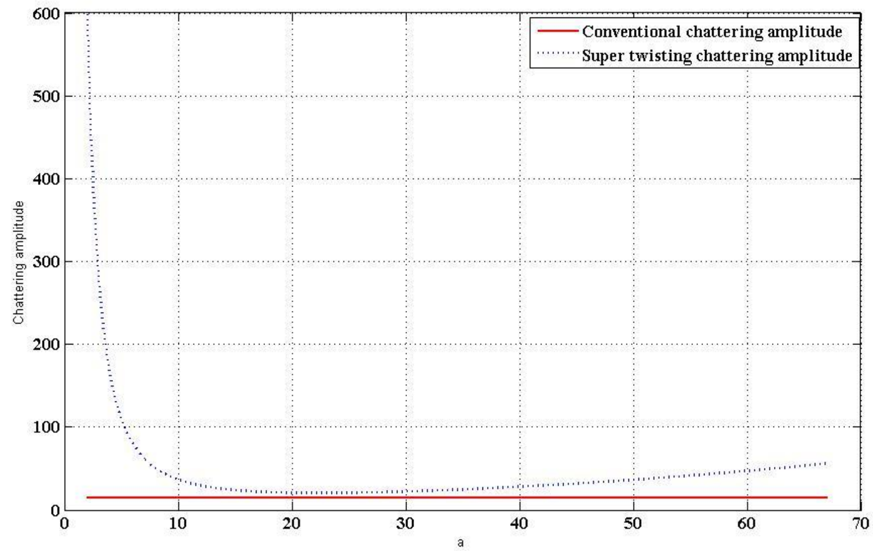

Figure 4.9 Chattering amplitude of CSMC and STSMC using DFM, $\mu = 0.2$

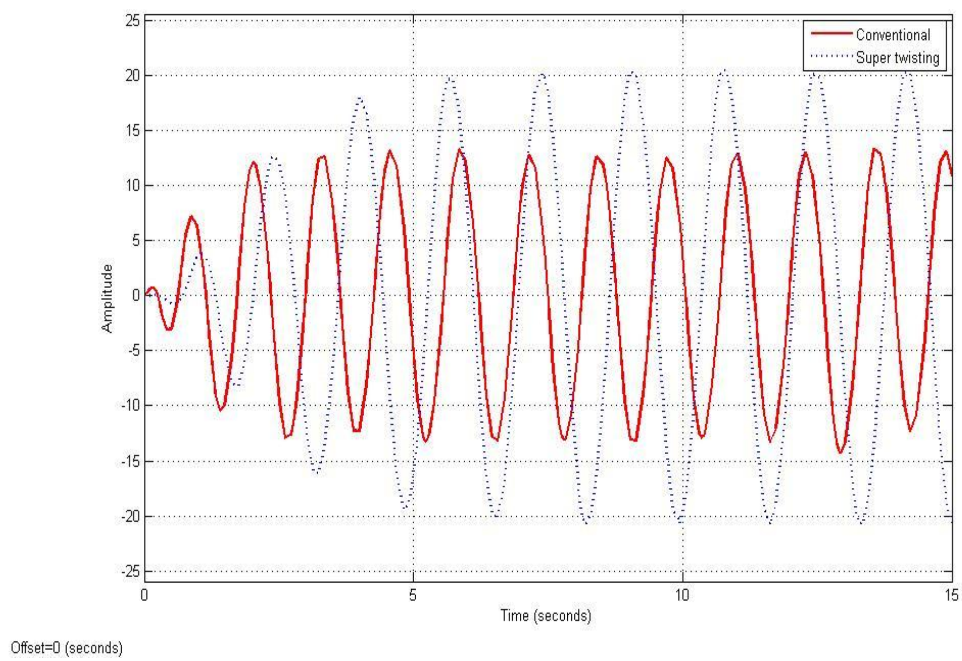

Figure 4.10 Chattering amplitude of CSMC and STSMC using simulation, $\mu = 0.2$

4.3 Part two: The disturbance $F \neq 0$

4.3.1 Analysis of conventional sliding mode control

Now same previous steps of analysis and simulation is performed, but only simulation is considered in the super twisting part, and that due the complication of applying the describing function method in this case.

First, the analysis is applied to the system with conventional sliding mode controller; the block diagram for the plant is shown in figure 4.8.

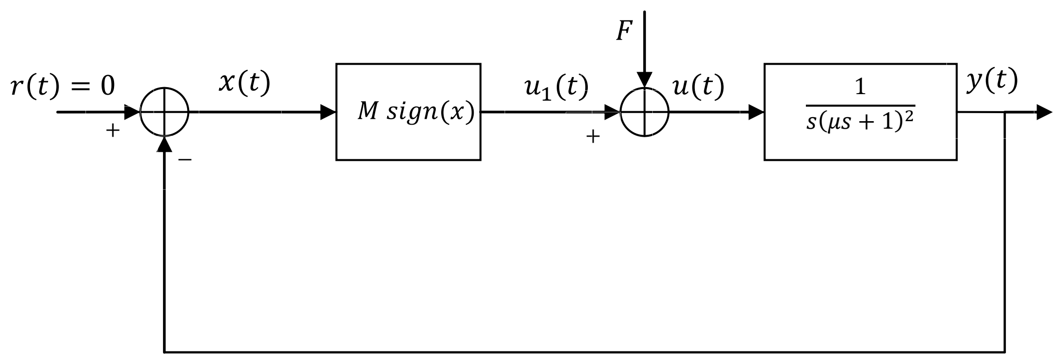

Figure 4.11 Simplified representation of STSMC scheme in the presence of the disturbance

The function $x(t)$ is considered to be $A_o + A_1 \sin(\omega t)$ and the output of the nonlinearity is approximated to be

$$u_1(t) = B_o + B_1 \sin(\omega t) + C_1 \cos(\omega t) \tag{4.31}$$

where

$$B_o = \frac{M\big((\pi+2\,z)-(\pi-2\,z)\big)}{2\,\pi} = -F \tag{4.32}$$

and

$$z = \sin^{-1}\left(\frac{A_o}{A}\right) \tag{4.33}$$

Substituting z back in equation (4.32) results in

$$B_o = \frac{2\,M\sin^{-1}\left(\frac{A_o}{A}\right)}{\pi} = -F \tag{4.34}$$

Continuing computing the other terms

$$B_1 = \frac{M}{2\pi}\left(\int_0^{\pi+2z} \sin(z)\,dz - \int_{\pi+2z}^{2\pi} \sin(z)\,dz\right)$$

$$C_1 = \frac{M}{2\pi}\left(\int_0^{\pi+2z} \cos(z)\,dz - \int_{\pi+2z}^{2\pi} \cos(z)\,dz\right) \tag{4.35}$$

The two previous integrals result in

$$B_1 = \frac{4\,M}{\pi}\left(1 - \frac{A_0^2}{A^2}\right)$$

(4.36)

$$C_1 = -\frac{4\ A_o\ M\ \left(1\ - \frac{A_o^2}{A^2}\right)^{\frac{1}{2}}}{A\ \pi}$$

Let $u(t)$ be the output of nonlinearity after canceling B_o with F, that is,

$$u(t) = \frac{4\,M}{\pi}\left(1 - \frac{A_0^2}{A^2}\right)\sin(\omega t) - \frac{4\ A_o\ M\left(1 - \frac{A_0^2}{A^2}\right)^{\frac{1}{2}}}{A\ \pi}\cos(\omega t) \qquad (4.37)$$

To find A and ω, we equate $\frac{u(t)}{A}$ to the inverse transfer function $g(j\omega)$ after changing the two function from the current form

$$y = \alpha \sin(\omega t) + \beta \cos(\omega t) \qquad (4.38)$$

to the form

$$y = \sqrt{\alpha^2 + \beta^2}\sin\,(\omega t + \varphi),\ \varphi = \arctan\,\left(\frac{\beta}{\alpha}\right) \qquad (4.39)$$

The equality between $\frac{u(t)}{A}$ and $g(j\omega)$ yields the next two equations

$$\sqrt{(2\mu\omega^2)^2 + (\mu^2\omega^3 - \omega)^2} = \sqrt{{16\, M^2\, (A^2 - \, A_o^2)}\Big/{A^4\, \pi^2}} \tag{4.40}$$

$$\left({\mu^2\omega^3 - \omega}\Big/{2\mu\omega^2}\right) = -\,{A_o}\Bigg/{A\, \left(1\, - \frac{A_o^2}{A^2}\right)^{\frac{1}{2}}} \tag{4.41}$$

Using equations (4.34), (4.40), and (4.41) yields

$$\omega_1 = \frac{\sqrt{\frac{1}{cos\left({F\,\pi}/{2\,M}\right)^2}}}{\mu} + {sin\left(\frac{F\,\pi}{2\,M}\right)}\Bigg/{\mu\,\sqrt{cos\left(\frac{F\,\pi}{2\,M}\right)^2}} \tag{4.42}$$

$$\omega_2 = {sin\left(\frac{F\,\pi}{2\,M}\right)}\Bigg/{\mu\,\sqrt{cos\left(\frac{F\,\pi}{2\,M}\right)^2}} - {\sqrt{\frac{1}{cos\left(\frac{F\,\pi}{2\,M}\right)^2}}}\Bigg/{\mu} \tag{4.43}$$

Each root of the frequency ω results in two roots of the amplitude A, that is,

$$A_{1,\omega_1} = \frac{4\,M\,\mu\,Q\text{^}3\,(\mathcal{P}_1)^{\frac{1}{2}}}{\pi}$$

$$(4.44)$$

$$A_{2,\omega_1} = -\frac{4\,M\,\mu\,Q\text{^}3\,(\mathcal{P}_1)^{\frac{1}{2}}}{\pi}$$

$$A_{1,\omega_2} = \frac{4\ M\ \mu\ \mathcal{Q}\text{^}3\ (\mathcal{P}_2)^{\frac{1}{2}}}{\pi}$$

(4.45)

$$A_{2,\omega_2} = -\frac{4\ M\ \mu\ \mathcal{Q}\text{^}3\ (\mathcal{P}_2)^{\frac{1}{2}}}{\pi}$$

The functions $\mathcal{P}_1$ and $\mathcal{P}_1$ are defined as

$$\mathcal{P}_1 = \frac{8\,\mathcal{Q}^2 - 8\,\mathcal{Q}^4 + \mathcal{Q}^6 + 8\,\mathcal{R}\left(\frac{1}{\mathcal{Q}^2}\right)^{\frac{5}{2}}(\mathcal{Q}^2)^{\frac{7}{2}} - 4\,\mathcal{R}\left(\frac{1}{\mathcal{Q}^2}\right)^{\frac{5}{2}}(\mathcal{Q}^2)^{\frac{9}{2}}}{\mathcal{Z}_1 + 320\,\mathcal{R}\left(\frac{1}{\mathcal{Q}^2}\right)^{\frac{5}{2}}(\mathcal{Q}^2)^{\frac{9}{2}} - 32\,\mathcal{R}\left(\frac{1}{\mathcal{Q}^2}\right)^{\frac{5}{2}}(\mathcal{Q}^2)^{\frac{11}{2}} + 512}$$

(4.46)

$$\mathcal{P}_2 = \frac{8\,\mathcal{Q}^2 - 8\,\mathcal{Q}^4 + \mathcal{Q}^6 - 8\,\mathcal{R}\left(\frac{1}{\mathcal{Q}^2}\right)^{\frac{5}{2}}(\mathcal{Q}^2)^{\frac{7}{2}} + 4\,\mathcal{R}\left(\frac{1}{\mathcal{Q}^2}\right)^{\frac{5}{2}}(\mathcal{Q}^2)^{\frac{9}{2}}}{\mathcal{Z}_2 - 320\,\mathcal{R}\left(\frac{1}{\mathcal{Q}^2}\right)^{\frac{5}{2}}(\mathcal{Q}^2)^{\frac{9}{2}} + 32\,\mathcal{R}\left(\frac{1}{\mathcal{Q}^2}\right)^{\frac{5}{2}}(\mathcal{Q}^2)^{\frac{11}{2}} + 512}$$

and the functions $\mathcal{Z}_1$, $\mathcal{Z}_2$, $\mathcal{Q}$ and $\mathcal{R}$ are given by

$$\mathcal{Q} = cos\left(\frac{F\ \pi}{2\ M}\right)$$

(4.47)

$$\mathcal{R} = sin\left(\frac{F\ \pi}{2\ M}\right)$$

$$\mathcal{Z}_1 = 640\, Q^4 - 1024\, Q^2 - 128\, Q^6 + 4\, Q^8 + 512\, \mathcal{R}\left(\frac{1}{Q^2}\right)^{\frac{5}{2}} (Q^2)^{\frac{5}{2}} - 768\, \mathcal{R}\left(\frac{1}{Q^2}\right)^{\frac{5}{2}} (Q^2)^{\frac{7}{2}}$$

(4.48)

$$\mathcal{Z}_2 = 640\, Q^4 - 1024\, Q^2 - 128\, Q^6 + 4\, Q^8 - 512\, \mathcal{R}\left(\frac{1}{Q^2}\right)^{\frac{5}{2}} (Q^2)^{\frac{5}{2}} + 768\, \mathcal{R}\left(\frac{1}{Q^2}\right)^{\frac{5}{2}} (Q^2)^{\frac{7}{2}}$$

It is noted from the equations (4.42), (4.43), (4.44), and (4.45) as μ tends to zero the frequency ω and the amplitude A tend to infinity and zero respectively, figures 4.12 and 4.13 show this behavior.

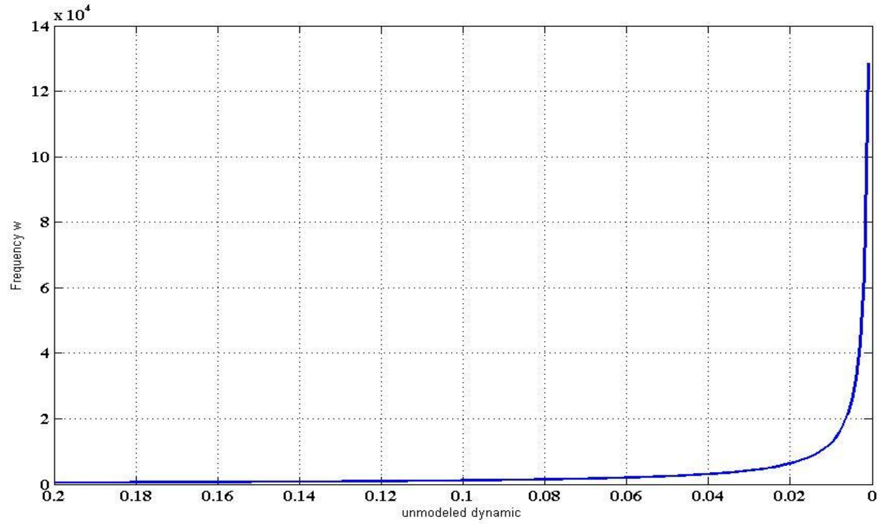

Figure 4.12 frequency (ω) VS the unmodeled dynamic μ for CSMC in the presence of the disturbance

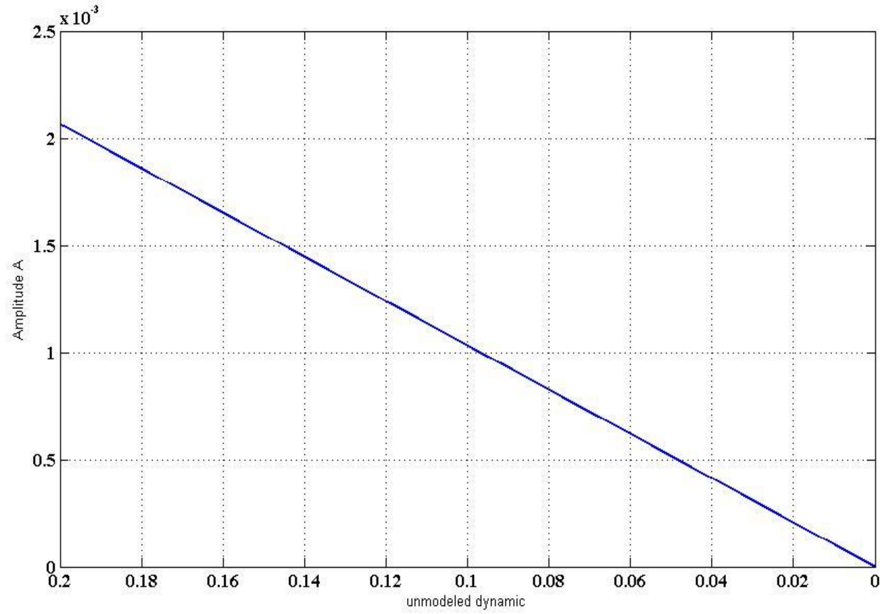

Figure 4.13 Amplitude (A) VS the unmodeled dynamic μ for CSMC in the presence of the disturbance

4.3.2 Simulation for conventional and super twisting methods

Next, the simulation is performed for conventional and super twisting technique in the presence of the disturbance F, where $F < |F_o|$ and F_o is known upper estimation of the disturbance. The parameter $M = \gamma F$ where $1.001 \leq \gamma \leq 1.1$. Figure 4.14 shows the two simulated systems. Noting that the parameters α in the super twisting algorithm is selected to maintain the stability of the system and in the same is minimized to result in the minimum value of chattering amplitude.

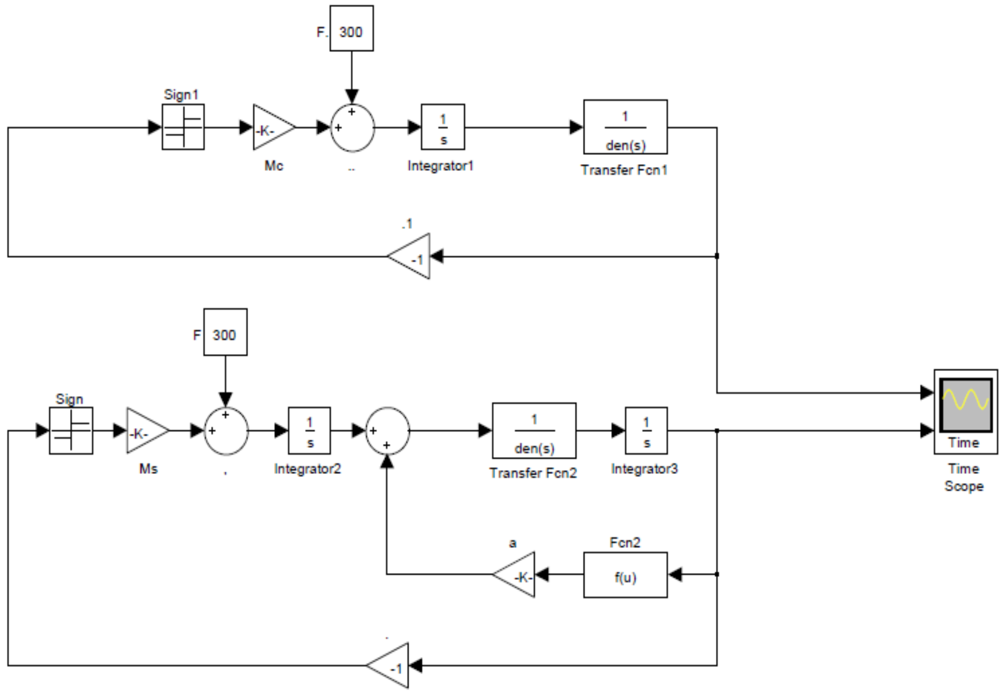

Figure 4.14 Matlab block simulation of CSMC and STSMC in the presence of the disturbance

Several cases are considered in the simulation as follow

Case 1: $\boldsymbol{\mu = 0.1, \gamma = 1.01}$

In this case, it is noted, from figures 4.15 and 4.16, that that amplitude of chattering when the conventional sliding mode control is used is less than the chattering amplitude in case of super twisting algorithm. This result is widely expected since super twisting technique has been addressed in several literatures as alternative solution of chattering problem in first order sliding mode control.

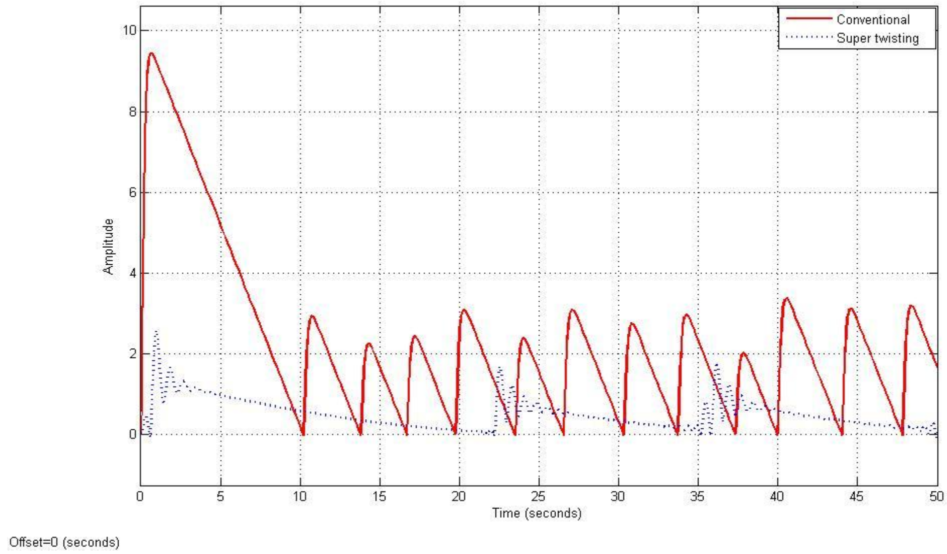

Figure 4.15 Chattering amplitude of CSMC and STSMC using simulation when the disturbance $F = 100$

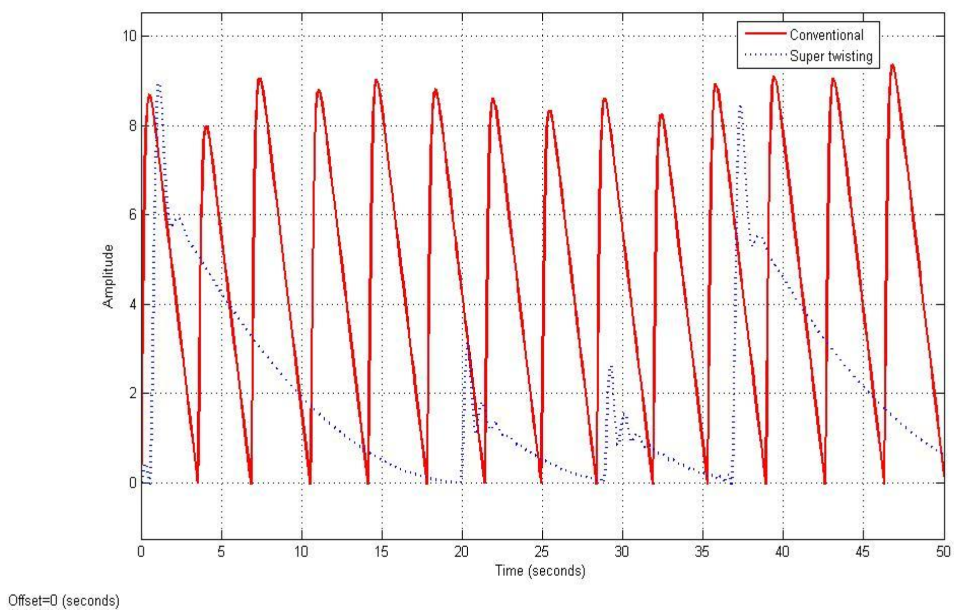

Figure 4.16 Chattering amplitude of CSMC and STSMC using simulation when the disturbance $F = 300$

Case 2: $\mu = 0.1, \gamma = 1.001$

Here, decreasing the value of γ contribute to decrease the chattering amplitude of the two methods. In addition, the two amplitude are nearly equal when the disturbance $F = 100$ as shown in figure 4.17. In the case in which $F = 300$ chattering amplitude of the conventional method is clearly less than the super twisting case as shown in figure 4.18.

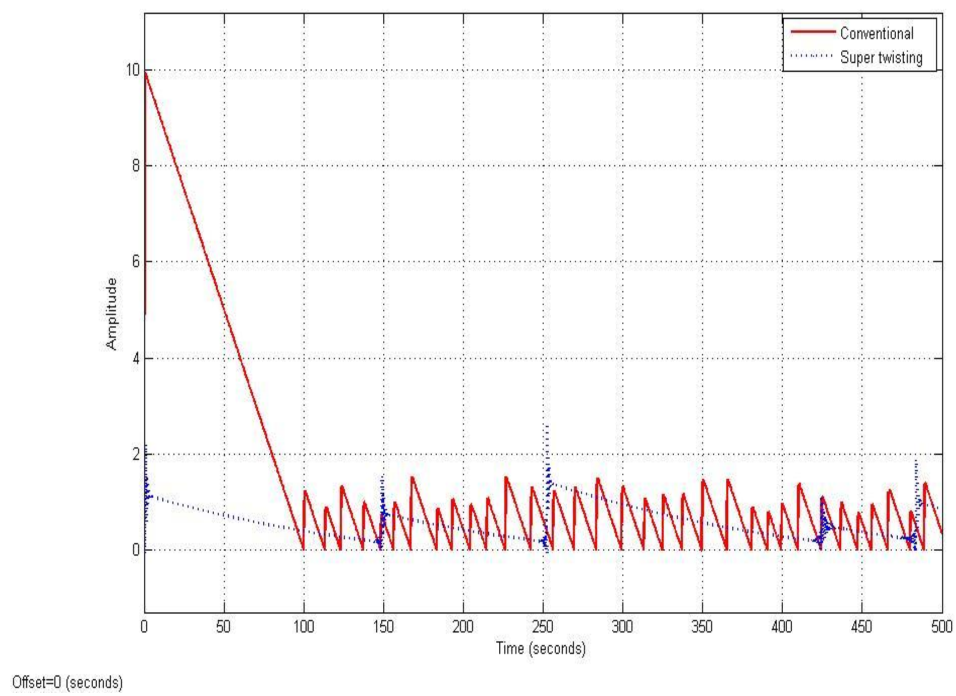

Figure 4.17 Chattering amplitude of CSMC and STSMC using simulation when the disturbance $F = 100$

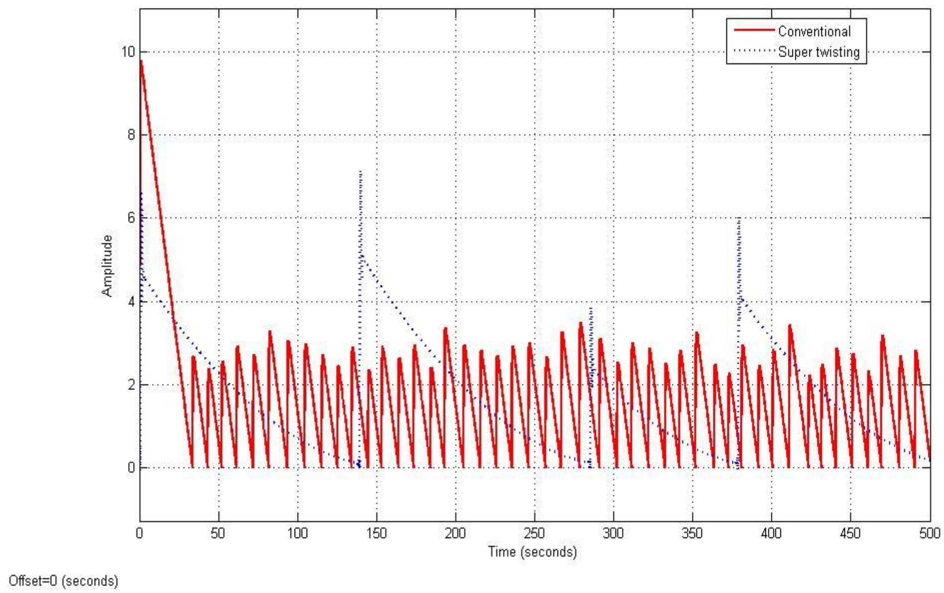

Figure 4.18 Chattering amplitude of CSMC and STSMC using simulation when the disturbance $F = 300$

Case 3: $\boldsymbol{\mu = 0.2, 1.001 \leq \gamma \leq 1.01}$

The situation here is different. Changing the value of γ does not change the fact that the chattering amplitude of the conventional controller is always and significantly higher than the amplitude of chattering of the super twisting algorithm regardless of the disturbance value. Figures 4.19 to show this behavior and two value of γ are considered**.**

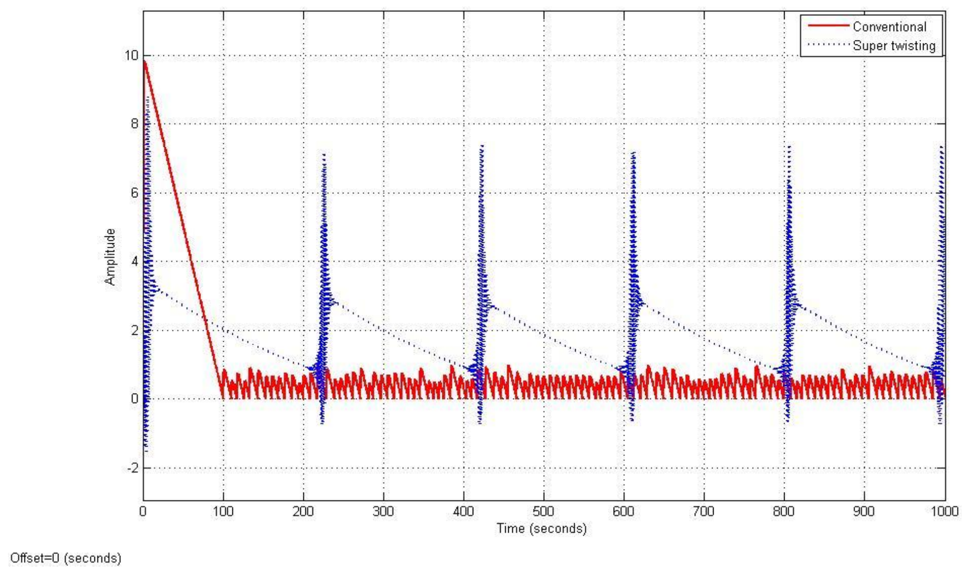

Figure 4.19 Chattering amplitude of CSMC and STSMC using simulation when the disturbance $F = 10, \gamma = 1.01$

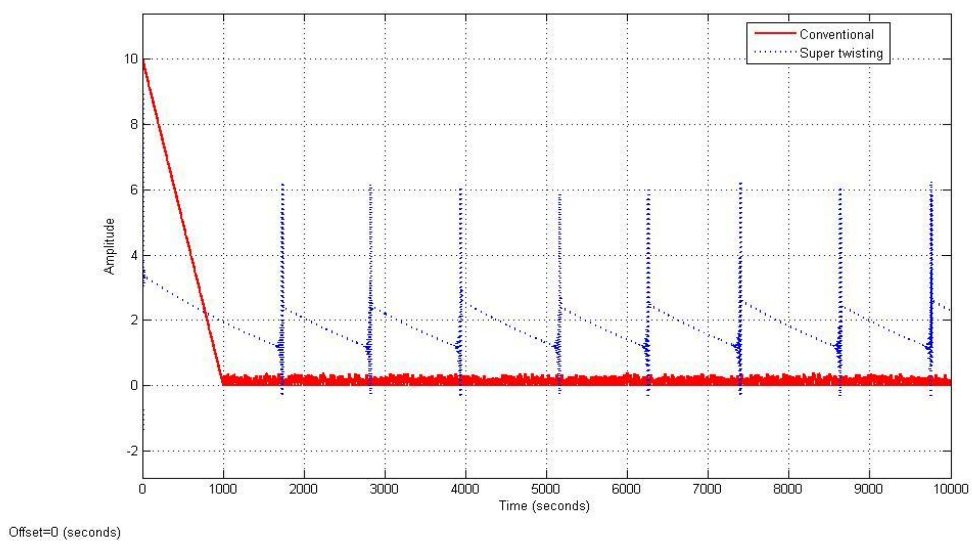

Figure 4.20 Chattering amplitude of CSMC and STSMC using simulation when the disturbance $F = 10, \gamma = 1.001$

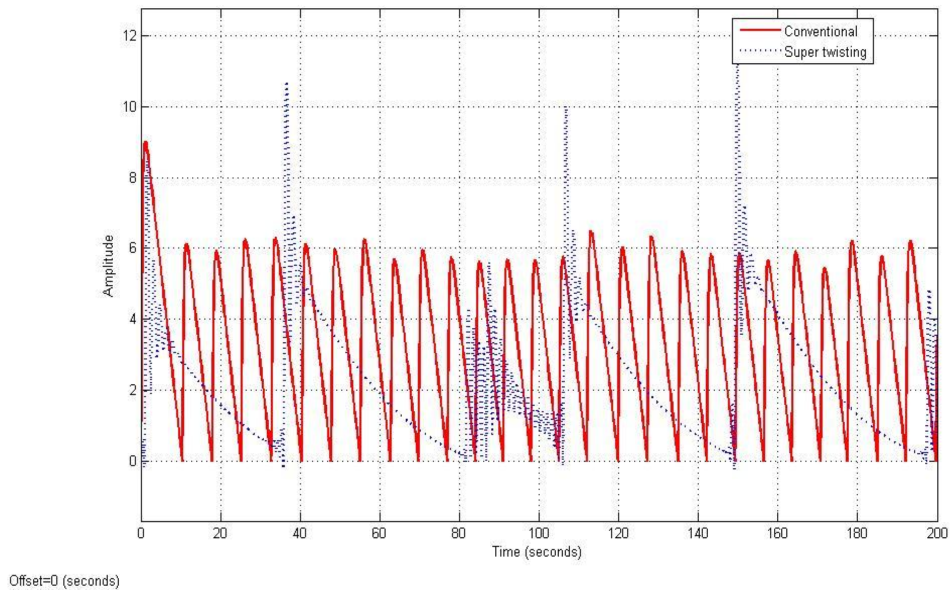

Figure 4.21 Chattering amplitude of CSMC and STSMC using simulation when the disturbance $F = 100, \gamma = 1.01$

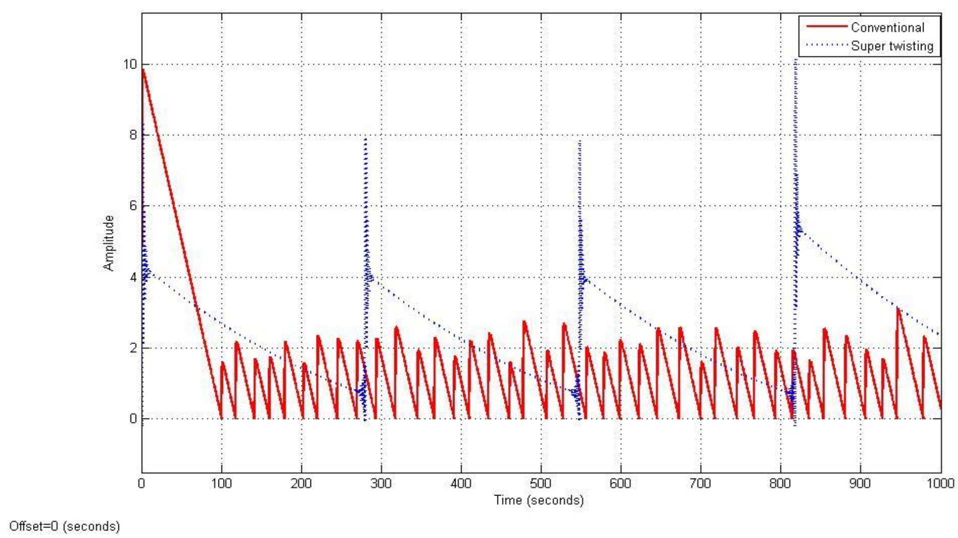

Figure 4.22 Chattering amplitude of CSMC and STSMC using simulation when the disturbance $F = 100, \gamma = 1.001$

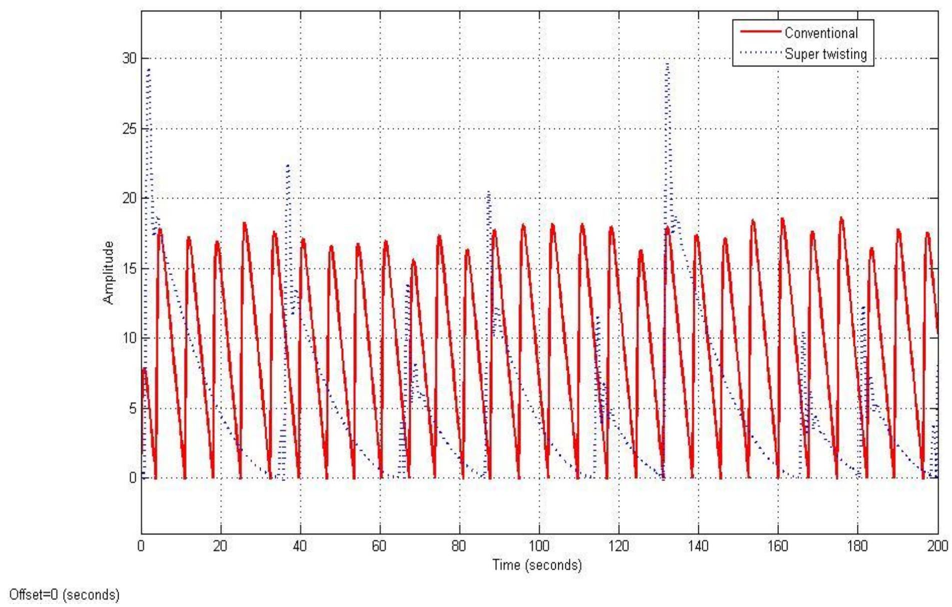

Figure 4.23 Chattering amplitude of CSMC and STSMC using simulation when the disturbance $F = 300$, $\gamma = 1.01$

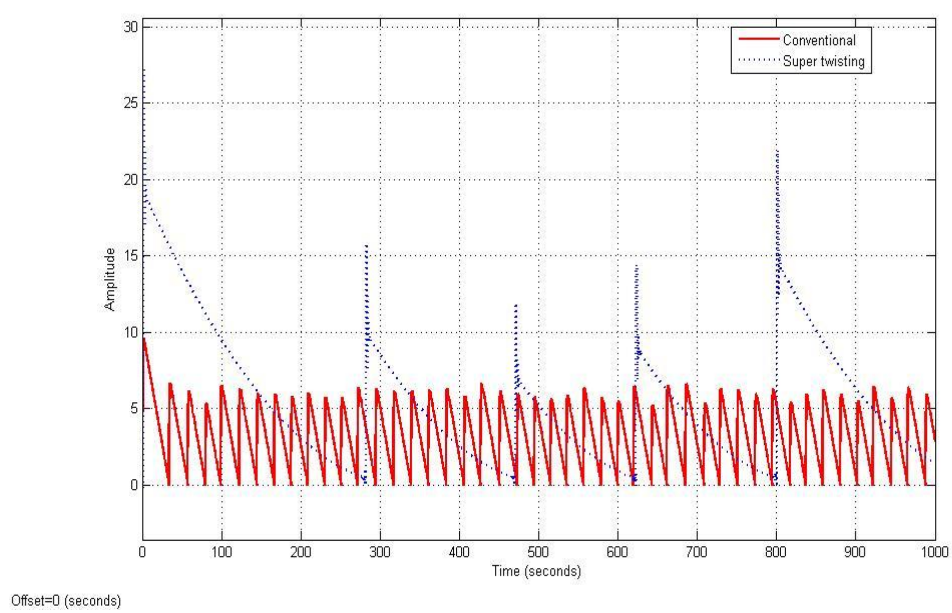

Figure 4.24 Chattering amplitude of CSMC and STSMC using simulation when the disturbance $F = 300$, $\gamma = 1.001$

Conclusion

In this thesis, several cases are considered to compare first order to second order sliding mode control. These cases confirm the claim that the super twisting algorithm is not always more satisfying solution than the conventional sliding mode control approach. Several parameters should be considered to decide which algorithm should be used. The value range of the unmodeled dynamic has a considerable effect of choosing the control algorithm. Despite the value of the disturbance F and if the value of the unmodeled dynamic $\mu = 0.2$ then ,without any debate, the first order sliding mode control is the appropriate to ensure having the minimum value of the chattering amplitude as in case 3. For the case in which $\mu \leq 0.1$, as long the value of the disturbance F increase the possibility of applying the conventional control is increase. In addition, an accurate choice of the function γ can reduce the chattering amplitude and even more makes the conventional control a better choice than the super twisting algorithm. The next table concludes these cases and gives the situation in which one method is more satisfying then the other.

	$F = 100$ $\gamma = 1.01$	$F = 100$ $\gamma = 1.001$	$F = 300$ $\gamma = 1.01$	$F = 300$ $\gamma = 1.001$
$\mu \leq 0.1$	super twisting	conventional	super twisting	conventional
$\mu = 0.2$	conventional	conventional	conventional	conventional

Table 1. Several cases show the more satisfying approach

The range of the disturbance F, the value of the unmodeled dynamic μ, and the related function γ between M and F are most important parameters effecting the amplitude of chattering. All these parameters should be considered to have a clear decision of which control method is more applicable.

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